Number Systems

There are more ways to represent numbers than the decimal system, and certain situations may call for other number systems instead of decimal if it's more convenient or efficient. Some representations which we'll go over include binary, octal, and hex.

Decimal System:

First, starting with decimal, our typically used number system is base-10, meaning we can use digits from 0 to 9. Each place on a number is multiplied by ten to an increasing exponent starting at zero for the least significant digit. For example:

$$123 = 100 + 20 + 3 = (1 * 10^2) + (2 * 10^1) + (3 * 10^0)$$

The same concept applies to other number systems, but where we will replace the 10 with other numbers. For binary, this means 2, for octal, this means 8, and for hex, this means 16.

Binary System:

So, now, consider binary. Instead of having access to digits from 0 to 9, we have access to 0 and 1. For example, we may have the number 1011 at hand. How can we convert this binary number to a decimal representation? You can multiply each digit out much like the above example; for each digit, multiply the 1 or 0 by 2 to the power of its index, starting with zero for the rightmost digit, then add the values together. So:

$$1011 = (1 * 2^3) + (0 * 2^2) + (1 * 2^1) + (1 * 2^0) = 8 + 0 + 2 + 1 = 11$$

Using this technique, we could see that 1011 in binary translates to 11 in decimal form.

Octal System:

Binary tends to refer to two possible options, and as we saw, in our number representation case that meant 0 or 1. Octal refers to eight, so as you might have guessed, octal works by representing numbers with the possible digits from 0 to 7! So, if we are given a number in octal representation and want to convert to decimal, we can again apply our technique of multiplying each digit by the amount of possible digits in that slot, so for octal, that means 8 to the power of whichever place we're at.

For example, let's say we're given the number 122, which we are told is in octal, and would like to convert this to decimal. How would we approach this?

$$122 = (1 * 8^2) + (2 * 8^1) + (2 * 8^0) = 64 + 16 + 2 = 82 \text{ in decimal}$$

Hexadecimal System:

Hex, or hexadecimal, will work exactly the same way, except with 16 possible digits! You might wonder, though - how can we represent 16 possible digits? Hex has solved this problem by allowing 0-9, which have the same representation we would expect in decimal, but ALSO allows for the characters A through F to be valid parts of a hex number. Specifically, here's what they each correspond to in decimal:

A: 10
B: 11
C: 12
D: 13
E: 14
F: 15

So, when converting to decimal, you would use these numbers they represent rather than A-F. If you've ever seen RGB color values, such as #FFFFFF, this happens to be a hex representation of the color.

Now that we have a general idea of what hex is, how can we convert hex values to their decimal representation? Going back to the representation of RGB color values, more specifically, the hex value in this case is actually representing three values; generally, RGB refers to (red, green, blue) representation of colors. Each of the colors can be represented as a number from 0 to 255. Knowing this, and knowing that FF is the highest number that can be represented, what would you guess the value #FFFFFF maps to?

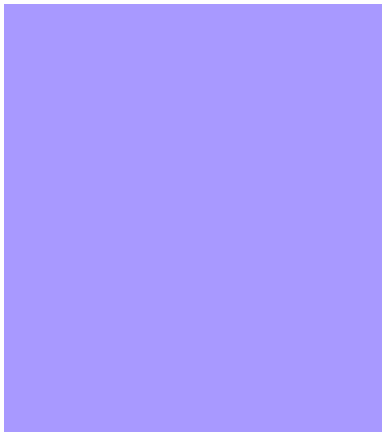
#FFFFFF maps to white, because it is representing (red, green, blue) as the values (255, 255, 255). Black would be represented by #000000 with the values (0, 0, 0), and all other colors fall as some value in between these two.

For example, what about #a799ff? To translate this into decimal, first, let's split it into three numbers. We are doing this due to the representation of three different colors, but if this were one big hex number, there would not be any splitting into different segments.

We find that red is represented by the number "a7." Blue is "99." Green is "ff." So, translating all of these to decimal:

$A7 = (10 * 16^1) + (7 * 16^0) = 160 + 7 = 167$
$99 = (9 * 16^1) + (9 * 16^0) = 144 + 9 = 153$
$FF = (15 * 16^1) + (15 * 16^0) = 240 + 15 = 255$

Translating to decimal, we find that this maps to (167, 153, 255) in decimal. What does this color really look like?



Shown above is what #a799ff represents; try putting this value into software such as MS Paint, or any color picker of your choice. Generally, RGB representation is one example of where hex is widely accepted as a standard over decimal! Using hex forms of colors is most common for digital platforms such as the web.

ASCII Representation

In many computers, all characters are actually represented as numbers following the ASCII convention (ASCII is definitely not the ONLY way to represent characters, but is very common). ASCII represents 256 characters in total including A-Z, a-z, 0-9, and more. Other characters such as *, +, and / are also represented, as well as different characters which mean various things but tend to end up equally invisible when displayed on certain computers.

For example, the decimal value 65 represents ‘A’ in ASCII, and the decimal value 48 represents ‘0’ in ASCII. For a more comprehensive table of which characters map to certain ASCII values, take a look at the links in the More Resources section of this guide. Note that, due to ASCII’s range being 256 different characters, its binary representation is typically given in 8 digits per individual ASCII character. When using 8 digits in binary, the highest possible value you can store is 255, and the lowest possible value is 0, which adds up to 256 different numbers!

So, if you’re approached with some mysterious numbers in any number system, it’s possible you’ve been given a puzzle and need to translate the numbers to whichever ASCII characters line up with them. For example, maybe you are given the following binary:

01000111 01101111 00100000 01001001 01101100 01101100 01101001 01101110 01101001 00100001

What does this mean? How does this correspond to a secret message? If you can find an ASCII lookup table which directly contains binary numbers and shows which characters they represent, converting this into ASCII text will be a straightforward task of looking up each binary number and writing down the character you find. If you don’t have a binary lookup table, you

might need to convert these values to decimal first, then see what the decimal values mean in ASCII according to the lookup table.

If we pull out the Wikipedia page on ASCII, we can look these up quite easily. We can see that:

01000111 lines up with ‘G’
01101111 lines up with ‘o’
00100000 lines up with a space, so we’ll say that’s ‘ ‘
01001001 lines up with ‘I’ (that’s a capitalized I, in case it’s ambiguous)
01101100 lines up with ‘l’ (that’s a lowercase L, not to be confused with capitalized I)
01101100 once again lines up with ‘l’
01101001 lines up with ‘i’
01101110 lines up with ‘n’
01101001 once again lines up with ‘i’
00100001 lines up with ‘!’

(Note that Wikipedia only shows 7 binary digits for each ASCII character and we have 8 shown. Just lop off the leftmost digit on our above example and proceed as normal.)

Put these characters together, and we end up seeing that all those 0s and 1s actually spell out the phrase “Go Illini!”.

Summarizing number system conversion

To do maybe: write pseudocode summarizing the conversion technique used here

New number value = 0
Current placement digit index = 0
For each digit in the original number, starting from right to left:

More Resources

- https://www.w3schools.com/colors/colors_picker.asp
- https://www.w3schools.com/charsets/ref_html_ascii.asp
- https://en.wikipedia.org/wiki/ASCII
- https://theasciicode.com.ar/
- https://www.mathsisfun.com/binary-number-system.html
- https://www.mathsisfun.com/hexadecimals.html

To learn more on how to convert from the decimal system to other number systems:

- https://www.tutorialspoint.com/how-to-convert-decimal-to-binary
- https://www.wikihow.com/Convert-from-Decimal-to-Binary
- https://www.tutorialspoint.com/how-to-convert-decimal-to-octal
- https://www.wikihow.com/Convert-from-Decimal-to-Octal

- https://www.tutorialspoint.com/how-to-convert-decimal-to-hexadecimal
- https://www.wikihow.com/Convert-from-Decimal-to-Hexadecimal